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Optical Faraday effect in germanium

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With 3 figures in the text

ABSTRACT

The Faraday rotation in doped and intrinsic single crystal germanium has been studied in the wavelength region 1.5–5.5 μ. The effect in n-type germanium, with carrier concentrations in the range 6×10^{16}–2×10^{18} cm^{-3}, is shown to consist of two essentially independent contributions, due to bound and free electrons. A value of the effective mass of conduction electrons was calculated from the free carrier rotation. The Faraday rotation due to bound electrons takes on very large values for wavelengths shorter than 1.6 μ and has an opposite sense to the rotation at longer wavelengths. In p-type germanium of impurity concentrations as high as 3×10^{17} cm^{-3}, no rotation was observed in the free carrier range.

Introduction

In recent years, there has been an increased interest in the application of magnetooptical methods to semiconductor problems. The measurement of Faraday effect belongs to this group of methods.

In a Faraday experiment, the crystal (substance) to be investigated is located in a homogeneous magnetic field, and its ability to turn the plane of polarisation of a traversing beam of radiation is determined. Usually the experimental arrangement is such that the beam is parallel to the magnetic field.

Measurements of the Faraday rotation in the infrared region have been used to obtain a value of the free electron effective mass in a few semiconductors. Such experiments have been made by Moss and collaborators, who have determined the effective mass of conduction electrons in InSb [1, 2], Ge [3], and GaAs [4]. These investigations have been carried out at wavelengths so long that the rotation is entirely due to free carriers. Few values have been reported of the Faraday rotation close to or inside the band gap region of semiconductors. As for intrinsic crystals, von Kimmel [5] has measured the rotation in GaAs, InP, GaP, and Si in a very limited wavelength region immediately outside the absorption edge.

In intrinsic semiconductors near the absorption edge the Faraday rotation is associated with the magnetic splitting of the energy bands. In doped crystals the rotation in this range is composed of the intrinsic rotation and the free carrier rotation. No explicit theoretical expression for the total rotation in this region has been published to date. A theory which were to predict the finer details of the wavelength dependence in the band gap region would have to take the detailed band structures into account.

In germanium, the main absorption edge corresponds to approximately 1.8 μ. The present investigation covers the wavelength region between 5.5 and 1.57 μ.

Experimental

The experimental arrangement consists of a light source (with chopper), polarizer, a germanium crystal in a homogeneous magnetic field, analyzer, monochromator, detector and a phase sensitive amplification system.

Source, detector and polarizers were chosen for optimum sensitivity in every wavelength range. Thus in the near infrared region a tungsten lamp and commercial infrared Polaroids were used with a PbS photoconductive cell as detector, whereas outside 2.5 μ a SiC rod, transmission type polarizers consisting of piles of polythene sheets [1, 2], and a liquid nitrogen cooled PbTe cell were used. Piles of AgCl plates were also tried as polarizers, but proved to cause more optical distortion than the two other types. The plane parallel germanium crystal disc was placed in the gap of an electromagnet, with its parallel surfaces perpendicular to the field direction. The radiation emerging from the crystal passed the analyzer; and a small Hilger monochromator with a quartz or a KBr prism selected the desired wavelengths.

The rotation of the plane of polarisation was measured in the following way. The intensity was recorded for different settings of the analyzer close to a minimum, the position of which was obtained by graphical evaluation. This procedure was then repeated with the direction of the magnetic field reversed. Thus the rotation was obtained as half the shift in the minimum position for the two field directions. All the experiments where made at room temperature.

Crystals[1]

Thickness and carrier concentration of the crystals used are collected in table 1. All the specimens have been cut perpendicular to a [111]-direction.

To attain the best compromise between large rotation and high transmittance in a certain wavelength interval, several specimens differing in thickness had to be used.

The carrier concentrations of the n-doped samples must be known accurately. They were determined from Hall and resistivity measurements on the same specimens as were used in the Faraday experiments. To find the carrier concentration, N, from the low field Hall coefficient, R_0, it is necessary to know the scattering mechanism. The case of ellipsoidal energy surfaces having over all cubic symmetry has been treated by Abeles and Meiboom [6]. For n-type germanium in the extrinsic range, the Hall coefficient may be written as:

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[1] The single crystals were delivered by the Institutet för Halvledarforskning (Hafo). The desired impurity content had been obtained by adding known amounts of antimony or indium to the germanium melt.

Table 1. Germanium specimens used in Faraday effect measurements.

Crystal no.	*Thickness cm*	*Carrier conc. cm*$^{-3}$
Ge 037 I 228 intrinsic	0.016	$<5\times10^{13}$
	0.025	
	0.055	
	0.116	
	0.185	
	0.458	
Ge 034 I Sb 116 *n*-type	0.071	6.38×10^{16}
	0.467	6.42×10^{16}
Ge 032 M8 Sb *n*-type	0.026	1.63×10^{18}
	0.102	1.68×10^{18}
	0.206	1.66×10^{18}
Ge 042 I In 30 *p*-type	0.067	$\sim3\times10^{17}$
	0.156	$\sim3\times10^{17}$
Ge 041 I In 29 *p*-type	0.060	$\sim3\times10^{16}$
	0.145	$\sim3\times10^{16}$

$$R_0=\frac{1}{N e c}\times r\times\frac{3K(K+2)}{(2K+1)^2}, \tag{1}$$

where K is the ratio of the longitudinal to the transverse effective mass, e the electronic charge, and c the velocity of light. The parameter r depends upon the relative occurrence of ionized impurity scattering and lattice scattering. With K calculated from cyclotron resonance data, the mass factor is 0.785, and the equation (1) becomes:

$$R_0=\frac{1}{N e c}\times r\times 0.785.$$

Since the carrier concentrations of our specimens are high, ionized impurity scattering cannot be neglected. Jones [7] has described a way to calculate r when the Hall mobility, R_0/ϱ, is known. However, his graph of r against ϱ^i/ϱ appears to be slightly in error for values of ϱ^i/ϱ smaller than 0.6. (ϱ is the measured resistivity and ϱ^i a theoretical resistivity calculated on the assumption that the ionized impurity scattering is the sole scattering mechanism.) A numerical evaluation of the integrals involved (loc.cit.) leads to $r_{\min}=1.03$ at $\varrho^i/\varrho=0.08$, rather than $r_{\min}=1.06$ at $\varrho^i/\varrho=0.2$. This result is also in good agreement with values tabulated by Lark-Horovitz [8].

The carrier concentrations of the p-type crystals were estimated from resistivity data. The mobilities were obtained from the values of Trumbore and Tartaglia [9], when ionized impurity scattering had to be taken into account.

Faraday rotation in intrinsic germanium

The measured wavelength dependence of the Faraday rotation in the intrinsic specimens is shown in fig. 1 and fig. 2. For these crystals, the rotation must be interpreted as a bound electron effect. For $\lambda\gtrsim3\,\mu$, the shapes of the curves can be accounted for by classical dispersion theory with the band to band tran-

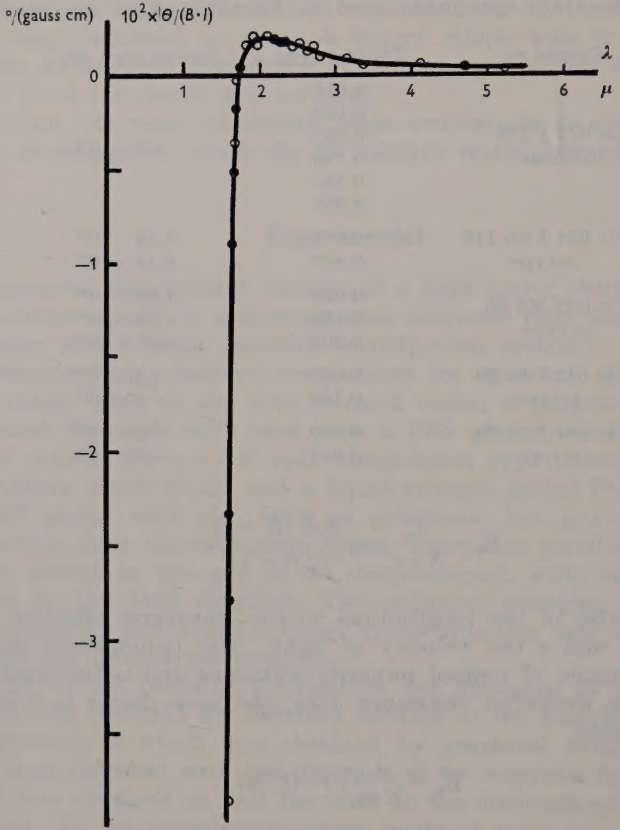

Fig. 1. Faraday rotation in intrinsic germanium.

sition represented by a single resonance frequency. The Faraday rotation can be related to the dispersion by means of the following equation (e. g. Moss [12]):

$$\frac{\theta}{l}=\frac{Be}{2cm^{*}}\times\lambda\frac{dn}{d\lambda}, \tag{2}$$

where B is the magnetic induction, e the electronic charge, λ the wavelength, c the velocity of light, m^* the effective mass, and θ/l the rotation per length of crystal.

The dispersion, $\frac{dn}{d\lambda}$, was calculated from values of the refractive index, n, given by Salzberg and Villa [10, 11]. By fitting the experimental values of the rotation to the equation (2), the value $m^*/m=1.7$ was found.

Similar calculations have been made by Moss [12] for ZnS and Se, using experimental values of the Verdet constant given by Becquerel. He obtained:

$m^*/m=1.1$ for ZnS, and $m^*/m=2.5$ for Se.

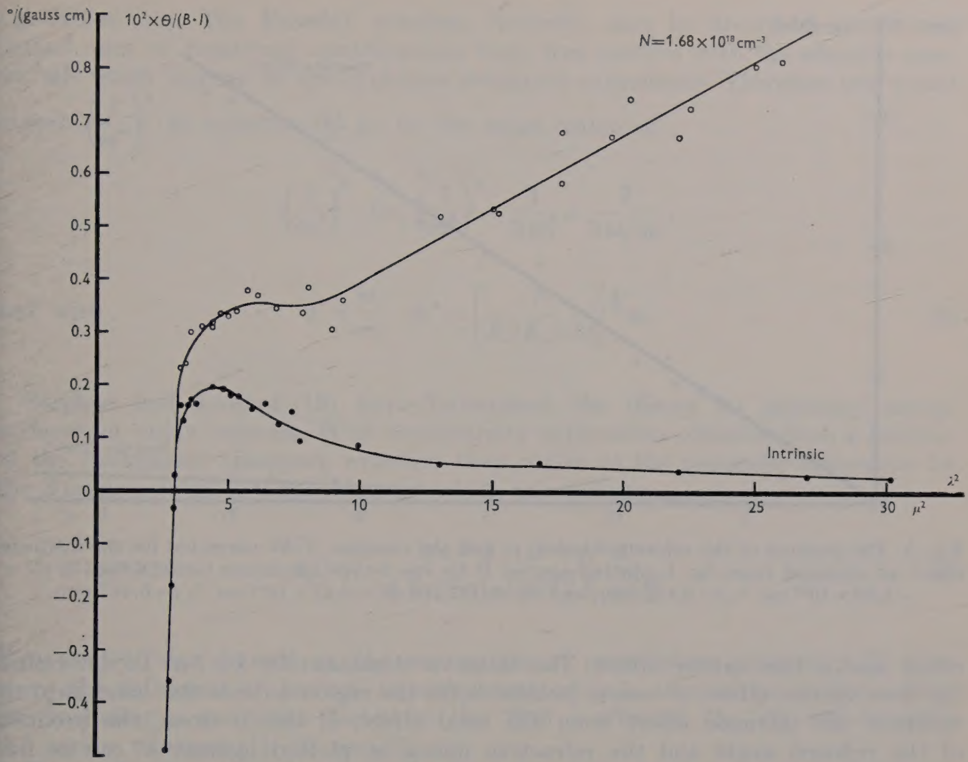

Fig. 2. Faraday rotation in intrinsic and in n-doped germanium, as function of the square of the wavelength. The curve for the n-doped germanium has been compiled from measurements on samples with $N = 1.68 \times 10^{18}$ cm^{-3}, $N = 1.66 \times 10^{18}$ cm^{-3}, and $N = 1.63 \times 10^{18}$ cm^{-3}. (Ge 032 M8Sb of table 1.) θ has been normalized to a concentration of $N = 1.68 \times 10^{18}$ cm^{-3}.

The relevance of these effective mass values is limited to cases where a model of bound vibrating electrons can be applied.

In the band gap region itself, the curve (fig. 1) has the general shape to be expected from a classical approximation, where the total inter-band transition is described by a single oscillator frequency. It does not seem justified, however, to apply this simplified theory, which entirely neglects the detailed structure of the energy bands. To our knowledge, no rigorous theoretical treatment valid in this region has yet been published. Stephen and Lidiard [13] have suggested that oscillations will occur in the Faraday rotation, corresponding to transitions between the Landau levels in the valence and conduction bands. No quantitative results are presented, however.

Faraday rotation in *n*-doped germanium

The Faraday rotation plotted against the square of the wawelength for highly doped n-type germanium ($N = 1.68 \times 10^{18}$ cm^{-3}) is shown in fig. 2. The appearance of this curve indicates that the effect is a superposition of the intrinsic

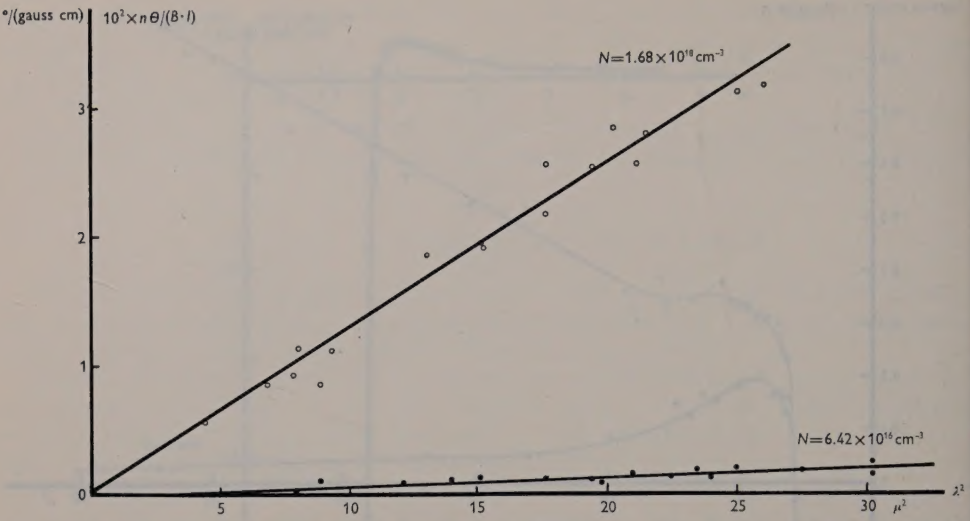

Fig. 3. The product of the refractive index, n, and the rotation, θ/Bl, corrected for the intrinsic effect as obtained from fig. 1, plotted against λ^2 for the n-type specimens Ge032M8Sb102 ($N = 1.68 \times 10^{18}\,\text{cm}^{-3}$; $t = 0.102$ cm), and Ge034ISb116 ($N = 6.42 \times 10^{16}\,\text{cm}^{-3}$; $t = 0.467$ cm).

effect and a free carrier effect. The latter is dominant for $\lambda \gtrsim 3\,\mu$. To determine the free carrier effect, it seems justifiable (in the region outside the band gap) to subtract the intrinsic effect from the total effect. If this is done, the product of the reduced angle and the refractive index, n, plotted against λ^2 can be fitted to a straight line (fig. 3). The refractive index, n, as determined by Salzberg and Villa [10, 11] has been corrected for free carrier dispersion.

Classical dispersion theory e.g. Mitchell [12], leads to the following expression for the free carrier Faraday effect at moderately long wavelengths:

$$\frac{\theta}{l} = \frac{e^3 B N \lambda^2}{8\pi^2 c^3 \varepsilon_0 n m^{*2}}, \tag{3}$$

where e is the electronic charge, B the magnetic induction, N the concentration of free carriers, λ the wavelength, c the velocity of light, ε_0 the vacuum dielectric constant, n the refractive index, m^* the effective mass, and θ/l the Faraday rotation per length of crystal.

In the case of more complicated energy surfaces, it is not immediately obvious what m^* represents in this expression. That may, however, be inferred from a consideration of cyclotron resonance theory. For conduction electrons in germanium the constant energy surfaces in k-space are ellipsoids of revolution oriented along the [111]-axes. The effective mass, m_i^*, which is determined in a cyclotron resonance experiment when the static magnetic field makes an angle α_i with the longitudinal axis of the energy surface, is:

$$\left(\frac{1}{m_i^*}\right)^2 = \frac{\cos^2 \alpha_i}{m_t^2} + \frac{\sin^2 \alpha_i}{m_t m_l} \tag{4}$$

e.g. Kittel [15]. The Faraday rotation, however, may be described as the statistical sum of rotational contributions from free carriers with the effective masses m_i^* which appear in the cyclotron resonance experiment. Therefore one would expect $\left(\frac{1}{m^*}\right)^2$ in equation (3) to be the mean value of

$$\left(\frac{1}{m_i^*}\right)^2; \text{ i.e. } \left(\frac{1}{m^*}\right)^2 = \frac{1}{3\,m_t^2} + \frac{2}{3\,m_t\,m_l},$$

and with

$$K = \frac{m_l}{m_t}, \quad m^* = \left[\frac{3}{K\,(K+2)}\right]^{\frac{1}{2}} m_l. \tag{5}$$

Stephen and Lidiard [13) have formulated the theory for arbitrary energy surfaces in cubic crystals. With conductivity expressions obtained from a solution of the Boltzmann transport equation they arrive at the particular expression for the Faraday rotation applicable to n-type germanium:

$$\theta/l = -\frac{3\pi e^2 K^2 (K+2)^2 B}{4 R_0 c^3 \omega^2 \varepsilon^{\frac{1}{2}} m_l^2 (2K+1)^2} \tag{6}$$

Equation (6) was derived on the assumption of pure lattice scattering. If, however, the influence of ionized impurity scattering is allowed for independently (by including a correction factor in the Hall coefficient R_0), (6) will comprise the earlier relations (1), (3), and (5).

From the slopes of the lines (fig. 3) and the measured values of the carrier concentrations, we have calculated the following values of the free electron effective mass:

$$m^*/m = 0.141 \qquad (N = 1.68 \times 10^{18}\ \text{cm}^{-3})$$

$$m^*/m = 0.12 \qquad (N = 6.42 \times 10^{16}\ \text{cm}^{-3})$$

The estimated errors in these determinations are five and ten per cent respectively.

Walton and Moss [13], under the more favourable conditions at longer wavelenghts (6–13 μ) determined the value $m^*/m = 0.135 \pm 0.004$. This is in close agreement with the value $m^*/m = 0.134$ which can be calculated from cyclotron resonance data. (The carrier concentration in their specimens was $1.47 \times 10^{17}\ \text{cm}^{-3}$.)

p-type germanium

A few sets of measurements were made on p-type crystals of carrier concentrations $3 \times 10^{16}\ \text{cm}^{-3}$, and $3 \times 10^{17}\ \text{cm}^{-3}$. Within the experimental accuracy, no free carrier rotation was observed. In any case the rotation due to free holes is much smaller than that due to the same number of free electrons. The reason for this is the larger effective mass of the holes in germanium.

Field dependence

The field dependence of the Faraday rotation was measured at some selected wavelengths. For intrinsic germanium measurements were made in the band gap region at $1.57\,\mu$, and outside this region at $2.0\,\mu$ and $2.2\,\mu$. For magnetic field strengths up to 7600 gauss, no deviation from linearity was observed in any of these cases. A corresponding measurement on the highly doped germanium was made at $4.7\,\mu$, where the free carrier effect is the dominant one. The expected linearity was confirmed.

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